Numerical Simulation of Internal Waves in the Littoral Ocean

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LONG-TERM GOAL

Our long-term goal is to employ numerical simulation to generate accurate predictions of nonhydrostatic internal-tide events, such as large internal waves and solitons, in the littoral ocean.

OBJECTIVES

Our oceanographic-scale objective is to work collaboratively with oceanographers carrying out field-scale experiments to quantify the significant wave events triggered by internal tides, including the nonhydrostatic formation of solitons and their evolution.

Our laboratory-scale simulation objective is to quantify the effects of the breaking instability as well as to study the three dimensional mechanisms of the breaking. These results will inform our field-scale efforts.

Our numerical objective is to blend a proven field-scale code with large-eddy simulation [LES] and the modeling of domains with irregular boundaries. Our tool is LES in three dimensions and time.

Our numerical analysis objectives include accurate representation of the flow near rough boundaries, creation of improved models for the sub-filter scale [i.e., unresolved] motions, and optimization of the computer code for multiprocessor computer systems.

APPROACH

For simulations of the littoral ocean, we will use a new, but proven, field-scale code [Casulli, 1999 a & b and Casulli and Walters, 2000]. It is a finite-volume, nonhydrostatic and free-surface code that employs an unstructured, staggered-grid. This code uses cells composed of Delaunay triangles in the horizontal plane with layers of uniform [but arbitrary] thickness in the vertical. The triangles allow boundary following in plan form, with a variable grid spacing so that one can concentrate grid points over canyons, etc. The thickness of the layers varies from layer to layer. We will install our large-eddy simulation [LES] subfilter-scale [SFS] model to handle turbulence; it [Katopodes, et. al, 2000 a & b] is based on what is called "velocity estimation".

A major focus of our work will be collaborating with oceanographers in dealing with the nonhydrostatic internal-tide events leading to large nonhydrostatic internal waves and solitons. We will examine the effect of three-dimensional bathymetry on internal tide generation, propagation, and transformation. Our code treats the entire domain and so the evolution from hydrostatic to nonhydrostatic motions is seamless.

Simultaneously, laboratory-scale simulation work is being carried out to examine in detail the physics of breaking internal waves and to test theory against repeatable laboratory experiments being done in the Environmental Fluid Mechanics Laboratory. The primary goal of this project is to quantify the mixing efficiency of laboratory-scale breaking interfacial waves in order to parameterize the effects for our larger scale models. The code used to model these waves employs many of the features of the Casulli code mentioned above.

WORK COMPLETED

In the past year, we have:

1. imported and made operational the simulation code to be used for field-scale simulations,
2. begun simulations of Monterey Bay tides with a first goal to reproduce the simulations of Rosenfeld, et al. (1999).
3. completed the subfilter-scale model development [Katopodes, et al., 2000 a & b].
5. made direct numerical simulations of breaking internal waves at the laboratory-scale.

RESULTS

Field-scale simulations: We have spent our time since the Casulli code was acquired disecting it and implementing it on our local processors. Great care was needed since this is an unstructured grid code written in FORTRAN90 and implementing many of its best features [including one which required our F90 software vendor to execute a bug fix!]. It is a very dense short code that is very fast. Currently, we are beginning runs to duplicate the simulations of Rosenfeld, et al. (1999), having obtained the topographic data from them. This is Major Task 1 of our work and still has a targeted completion date of Spring 2001.

Laboratory-scale simulations: Our significant results to date are from the laboratory-scale effort. The main objectives of this research have been to quantify the mechanisms that lead to breaking of interfacial waves away from boundaries as well as to quantify the mixing efficiency at the laboratory scale, in order to apply the knowledge to larger scale models by parameterizing the mixing and dissipative processes that occur at scales too small to realistically resolve numerically. Fringer, et al. (2000) report on this work.

Last year's progress report included results from simulations of interfacial waves breaking on sloped topography. While a great deal of insight into the dynamics of breaking can be attained through these simulations, the dynamics of these waves is a strong function of the topography itself. A literature review, however, showed that little work had been done with regard to breaking progressive interfacial waves away from boundaries. A reason for this may be the difficulty of generating large amplitude interfacial waves not only experimentally in the laboratory, but also numerically. Currently, Troy and Koseff (2000) are working on generating interfacial breaking waves away from boundaries experimentally. They focus several small amplitude interfacial waves to a point such that their interaction generates a large amplitude interfacial wave that breaks and mixes the thin interface separating the upper and lower fluids. Theirs is the only known experimental work of this type.

Large amplitude breaking interfacial waves away from boundaries represent a base case from which to extrapolate a great deal of information about the mechanisms of the breaking process, as well as the nature of the breakdown from internal wave energy to turbulence. Quantification of the breaking process is made simpler by studying breaking interfacial waves away from boundaries because the geometric parameter space is limited to a very narrow set of only three variables: 1) wave amplitude, 2) wave length, and 3) interface thickness. Moreover, interfacial waves themselves are more suitable for a baseline study than waves in a continuously stratified medium because their dispersive character depends solely on the density difference between the two layers and not on the actual density distribution, which can affect both the speed and direction of internal wave propagation.

In order for an interfacial wave to break, its steepness must exceed some critical value so that an instability occurs. What exactly triggers this instability is one of the objectives of this research, since evidence in the literature is contradictory. For example, Holyer (1979) predicts that Boussinesq interfacial waves, or waves with small density differences between the two layers, develop a verticality halfway between the crest and trough at which the fluid velocity within the wave exceeds its phase speed . When this occurs, a convective instability develops that leads to breaking.Tsuji and Nagata (1973), however, contend that interfacial waves will break due to a Kelvin-Helmholtz instability that results from excessive shear between the upper and lower layers long before the amplitude reaches the critical convective amplitude of Holyer. Thorpe (1978) shows that both can happen. But in his interfacial wave experiments, he imposes a background shear, which becomes the deciding factor in the breaking dynamics. Unlike Thorpe's waves and waves breaking on sloped topography, the numerical experiments we perform generate breaking waves whose instabilities are triggered by the velocity field induced by the wave field itself. As a result, our waves break of their own accord.

Because the interfacial breaking waves in which we are interested are Boussinesq in nature, the steepness at which they break is much higher than surface waves. Holyer (1973) predicted that Boussinesq waves break at a steepness of ka ~ 1.1, where k is the wavenumber and a is the wave amplitude. As a result of this steepness (surface waves break when ka ~ 0.11), interfacial breaking waves are highly nonlinear in nature and extremely difficult to initialize in a numerical solution. Rather than attempting to initialize the solution with an interfacial wave just before it is breaking, we initialize or force the solution with small amplitude waves, and then use some mechanism to force them to grow and become unstable. We have implemented two methods to generate breaking interfacial waves.

Breaking Technique 1: Baker, et. al, 1982, imposed a pressure field at the free surface and moved it at the phase speed of a surface wave, causing it to grow in amplitude until it breaks. Rather than applying it at the surface, we impose a pressure at the interface, integrate over the depth and include it in the Navier-Stokes equaitons as a horizontal momentum source. Because the phase speed is a function of the amplitude of the wave being forced, we track the interface profile to keep the momentum source in phase with the wave as it accelerates. The most significant advantage of this generation method is that it enables the use of a periodic domain so that only one wave is resolved by the available discretization. As a result, the number of grid points per wavelength is maximized. A typical time series of the breaking process is shown in Figure 1.

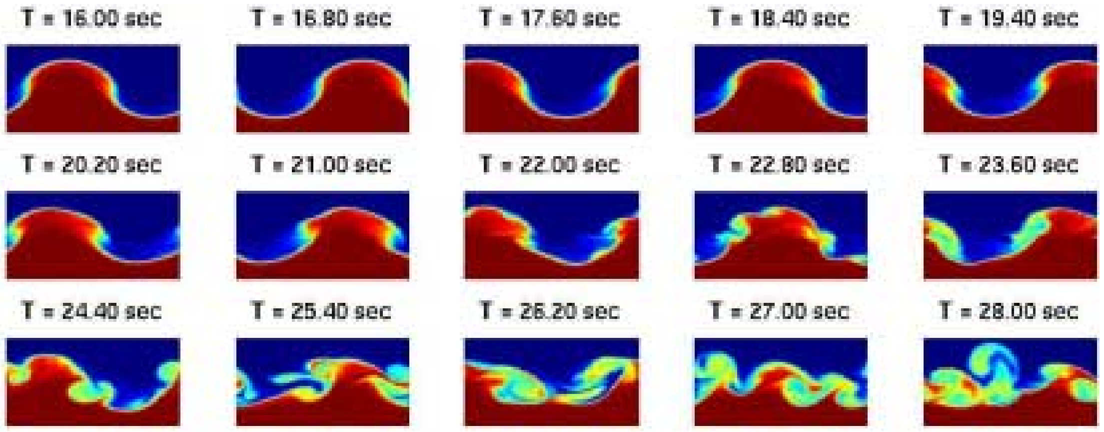

Figure 1. Breaking interfacial wave with a steepness of ka=1.1 generated with a moving momentum source.

In this figure, a 20 cm long interfacial wave with a density difference of 30 kg/m^3 and an interface thickness of 1 cm is shown breaking due to a convective instability in which the fluid velocity half way between the crest and trough exceeds the phase speed of the wave. At T=20.20 seconds, the red heavy fluid is descending into the blue fresher fluid due to convection in a Rayleigh-Taylor instability. Soon after, however, Kelvin-Helmholtz billows develop at the interface as a result of the weakened stratification.

Breaking Technique 2: In our second method of generating large amplitude interfacial waves, we introduce small amplitude interfacial waves at the left end of the computational domain, and then reduce the density difference between the two layers as a function of horizontal distance from the source. As the wave propagates into the region of weaker stratification, its phase speed decreases in proportion to the decreased density ratio and its steepness grows, while its wavelength decreases to account for the lowered reduced gravity, as shown in Figure 2. It would not be possible to force such steep waves at the outset near x/L=0, because the momentum source would just mix the two layers without generating interfacial waves.

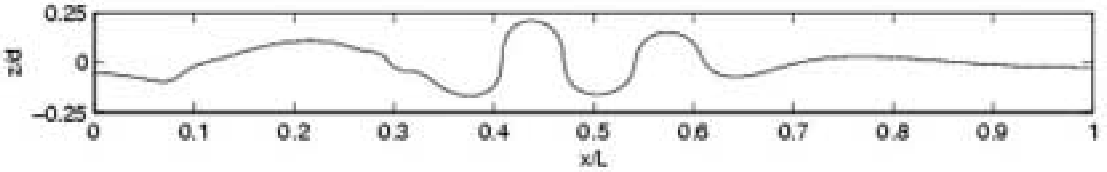

Figure 2. Effects of varying the stratification longitudinally, showing the decrease in wavelength after two waves have entered the weakly stratified region. The density ratio decreases from 30 kg/m^3 to 7.5 kg/m^3 between x/L=0.3 and x/L=0.5, where L=1.0 m, d=0.2 m, and the resolution is 256 by 80.

When the density difference is reduced to 7.5 kg/m^3, the wave steepness grows to 1.86 and the wave breaks due to nonlinear overturning, as shown in Figure 3. This steepness is substantially greater than that for the moving pressure field method, but this discrepancy is likely due to the generation mechanism, which is dependent upon the length scale over which the density ratio decreases horizontally. Because this length is on the order of the wavelength of the incipient wave, the leading edge of the wave encounters a reduced celerity before the trailing edge, and hence contributes to the steepening mechanism, in much the same way as an interfacial wave breaks as it encounters sloping topography. This could be remedied by increasing the size of the domain and slowly varying the horizontal density difference, but this would require more computation in order to maintain the same resolution. Nevertheless, the interfacial waves in Figure 3 demonstrate clearly that interfacial waves can break forward due to a convective nonlinearity in the absence of topographic effects before a shear instability develops.

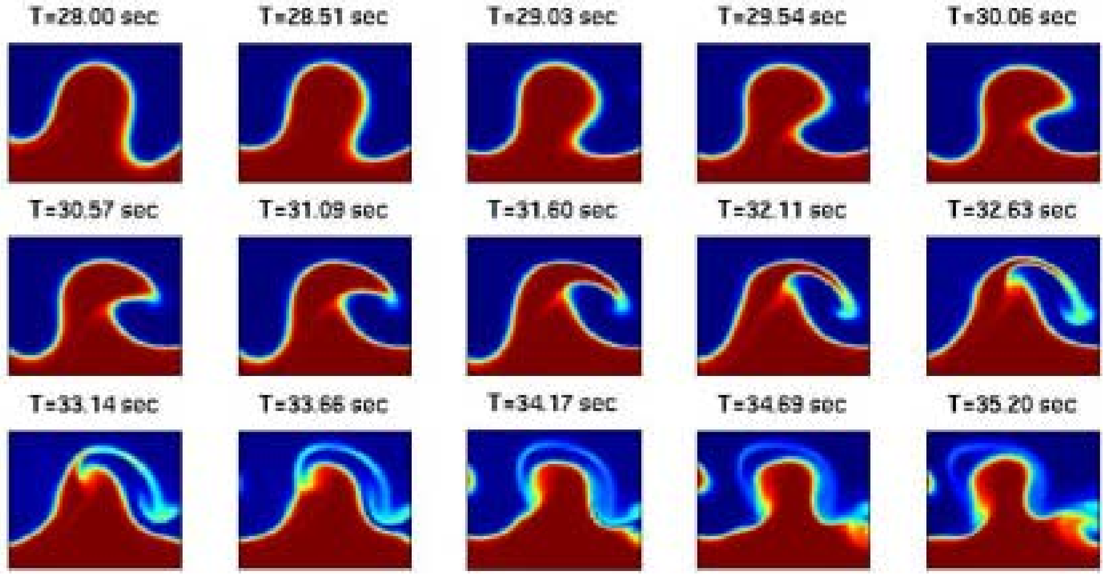

Figure 3. Breaking interfacial wave with a steepness of 1.86 generated by decreasing the density ratio in the horizontal. This wave is 15 cm long and the density difference between the two layers is 7.5 kg/m^3. The interface thickness is 1 cm.

In summary, we have developed two methods of generating large amplitude interfacial waves. Both methods show conclusively that interfacial waves can break forward due to a convective nonlinearity before a shear instability occurs. However, the first method has an advantage over the second because it can be performed in a periodic domain, and hence the breaking process can be resolved more accurately because we can perform simulations with one wave in the domain. Furthermore, it does not introduce a length scale associated with the decrease in the density ratio horizontally. Our plan is to perform three dimensional simulations using the moving momentum source method over a parameter space that will give us a fundamental understanding of the dynamics of a breaking interfacial wave in the absence of topographic effects.

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