Numerical Simulation of Internal Waves in the Littoral Ocean

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LONG-TERM GOAL

Our long-term goal is to employ numerical simulation to generate accurate predictions of the nature and behavior of high frequency, nonhydrostatic internal waves in the littoral ocean.

OBJECTIVES

Our oceanographic objective is to work collaboratively with oceanographers carrying out field-scale experiments to quantify the significant wave events triggered by internal tides, including the nonhydrostatic formation of solitons and their evolution [see, e.g., Petruncio, et al., 1998; Stanton and Ostrovsky, 1998].

Our numerical objective is to blend a proven field-scale code with our abilities in large-eddy simulation [LES] and the modeling of domains with irregular boundaries. Our tool will be LES in three dimensions and time.

Our numerical analysis objectives include accurate representation of the flow near rough boundaries, creation of improved models for the sub-filter scale [i.e., unresolved] motions, and optimization of the computer code for multiprocessor computer systems.

By carrying out simulations of waves breaking on slopes and comparing them to experiments being done by others in our laboratory, we will achieve a laboratory-scale simulation objective of quantifying the effects of the breaking instability as well as studying the three dimensional mechanisms of the breaking. These results will inform our field-scale efforts.

APPROACH

For simulations of the littoral ocean, we will use a new, but proven, field-scale code [Casulli, 1999 a, b & c]. The FORTRAN 90 code is fully developed; it has been used for estuaries so irregular boundaries, shallow to deep bathymetries, and open boundaries have been addressed. It is an unstructured, staggered-grid, finite-volume, fully-nonhydrostatic and free-surface code. Stanton and Ostrovsky (1998) observed a clear free-surface signature of their internal waves and Nagaosa (1999) discusses the importance of the free-surface treatment. This code uses cells composed of Delaunay triangles in the horizontal plane with layers of uniform [but arbitrary] thickness in the vertical. The triangles allow boundary following in plan form, with a variable grid spacing so that one can concentrate grid points over canyons, etc. The thickness of the layers varies from layer to layer. The code also treats the bottom depth variations in each cell correctly [cf., Chan and Street (1970)]. We will install our large-eddy simulation [LES] subfilter-scale [SFS] model to handle turbulence; it [Katopodes, et. al, 1999] is based on what is called "velocity estimation" [see, e.g., Domaradzki and Loh (1999)]. Our model is an exact solution of the partial differential equations for the evolution of the unresolved stresses and fluxes to 4th or 6th order in filter size.

A major focus of our work will be collaborating with oceanographers in dealing with the nonhydrostatic internal-tide events leading to large nonhydrostatic internal waves and solitons. We will examine the effect of three-dimensional bathymetry on internal tide generation, propagation, and transformation. Our code treats the entire domain and so the evolution from hydrostatic to nonhydrostatic motions is seamless.

The field-scale simulation work is being carried out by Mrs. Jiahe [Jan] Wang [Ph.D., Hohai U.]. She is a Ph.D. candidate and research assistant at Stanford.

Simultaneosly, laboratory-scale simulation work is being carried out by Mr. Oliver Fringer [BS: Princeton U.; MS: Stanford]. He is a Ph.D. candidate and research assistant at Stanford; he is supported by a DOE fellowship. This work allows us to examine in detail the physics of breaking internal waves and to test theory against repeatable laboratory experiments being done in the Environmental Fluid Mechanics Laboratory. The primary goal of this project is to quantify the mixing efficiency of laboratory-scale breaking interfacial waves in order to parameterize the effects for our larger scale models.

The code used to model these waves employs many of the features of the Casulli code mentioned above. The code splits the pressure into its hydrostatic and hydrodynamic components and can be used either as a hydrostatic or a nonhydrostatic code, since the hydrodynamic pressure is merely a correction to the dominant hydrostatic component of the flowfield. The hydrostatic component consists of the barotropic and the baroclinic components of the pressure, and because of the high speed of the barotropic waves, we model them implicitly with a theta-method. The hydrodynamic component of the flowfield is computed with a multigrid method.

WORK COMPLETED

This project began in March 1999. To date we have:

1. selected the simulation code to be used for field-scale simulations,
2. identified a bathymetry data set for Monterey Bay which has adequate resolution for our simulation [down to 5 meter increments in the horizontal],
3. identified grid-generation software for our triangular prism code elements [the assistance of the Coastal Division of the German Federal Waterways Engineering and Research Institute is facilitating this work].

4. advanced the subgrid-scale model development [Katopodes, et al., 1999].
5. examined the physics of breaking internal waves by laboratory-scale simulations.

RESULTS

At this early stage of the project, our results are from the laboratory-scale effort. Our current simulations demonstrate the three dimensional instabilities present during an interfacial wave breaking on a slope. Figure 1 depicts a 0.1 Hz wave propagating at a density interface in a two layer fluid and towards a slope of 11° in a 4 m tank with an interface thickness of 1 cm and water depth of 60 cm.

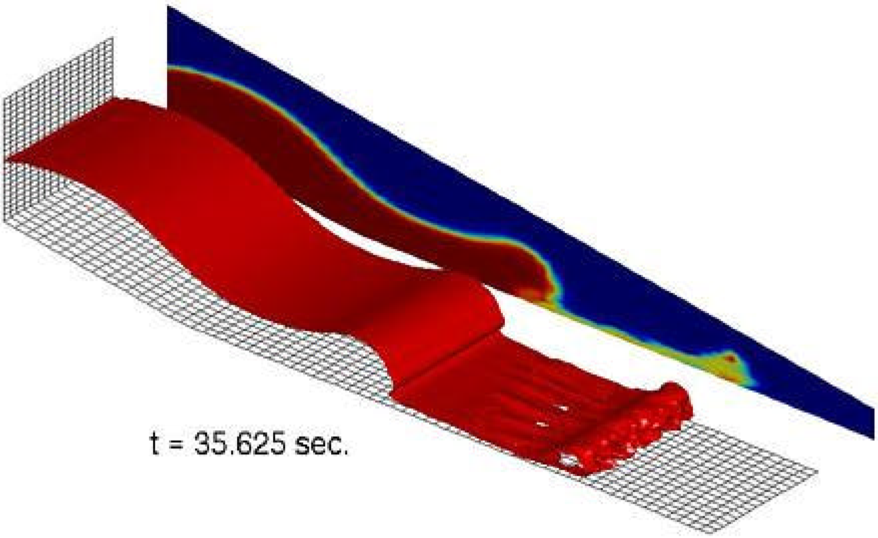

Figure 1. Breaking interfacial wave showing the longitudinal vortices and the Rayleigh-Taylor billows present after the first wave has broken.

The first wave has already broken and perturbed the flow enough so that the downslope current caused by the second wave can be seen breaking up into longitudinal rolls. These rolls are responsible in a large part for the mixing and dissipation resulting from an interfacial breaking wave. Figure 2 depicts a side view of the breaking event just as the first incipient wave is overturning on the slope.

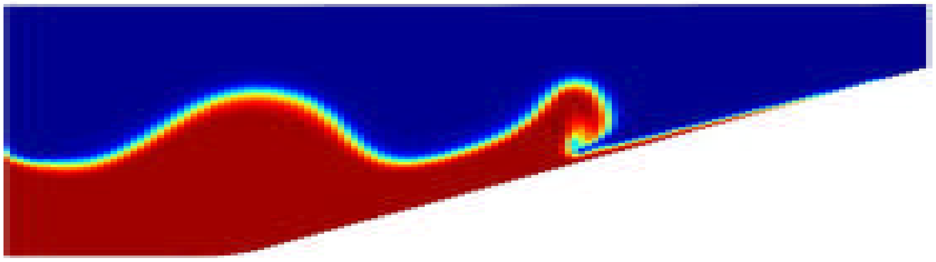

Figure 2. Side view at tank midplane showing the interfacial breaking wave just as it begins to overturn.

Unlike surface breaking waves, interfacial waves bulge at the crest as they break due to the shear at the interface between the two fluids. Therefore, the overturning mechanism is not as sharp, and leads to a secondary backward overturn due to the Kelvin-Helmholtz instability resulting from the shear stress at the interface that overtakes the forward overturning nonlinearity. This is shown in Figure 3, where the

Kelvin-Helmholtz instability is generating a backward billow after the forward overturn has subsided.

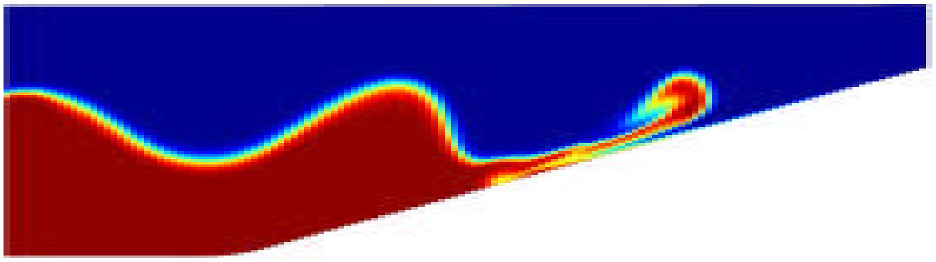

Figure 3. Side view at tank midplane showing the interfacial breaking wave as the secondary instability generates a backward-breaking Kelvin-Helmholtz billow.

While the two dimensional instabilities initiate the breaking event, the energy dissipation arises from its strong three dimensionality. We study the interfacial wave energy budgets by computing the energy history of an interfacial solitary wave breaking on a slop, as shown in Figures 4a and 4b.

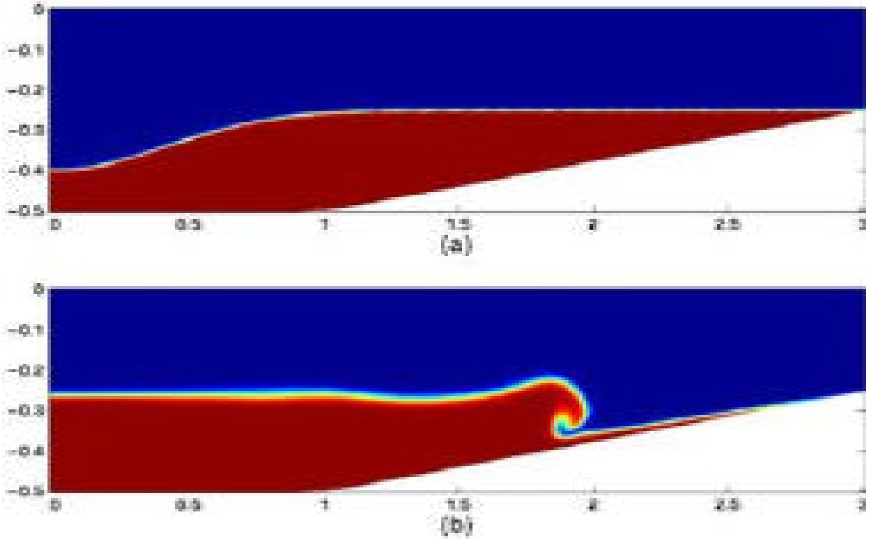

Figure 4. (a) Initial salinity distribution for the generation of a solitary wave. (b) Solitary wave breaking on a slope at t=18.0 seconds.

The solitary wave provides a much easier standpoint from which to measure the energy budget because the initial potential energy of the flowfield can be computed exactly. With this in mind, a comparison of the energy budgets for two and three dimensional flows is shown in Figure 5.

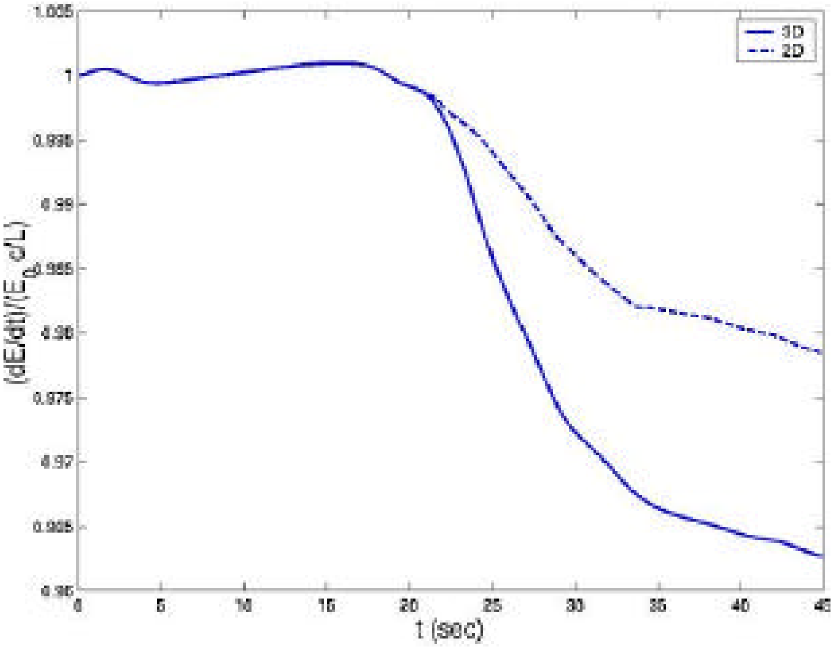

Figure 5. Total energy budget for a two and three dimensional solitary wave breaking on a slope.

The point where the solid line drops below the dashed line is where the three dimensionality of the flow takes over the dissipation. Roughly from t=25 to t=35 seconds energy for the three dimensional case is being dissipated via longitudinal vorticity dissipation. More energy is dissipated in this case than it is for the two dimensional case because the two dimensional case relies only on vertical and longitudinal shear to dissipate energy. Thereafter, the two and three dimensional flows match dissipation rates after the longitudinal vorticity has subsided.

Currently we are working on the simulation of breaking waves via characteristic focusing. These waves break as a result of the superposition of a train of waves that cause the amplitude of the wave to increase to a point where the fluid velocity beneath the crest is greater than the phase speed of the wave, thus causing it to overturn without the effects of bottom topography. Our interest in these waves arises from the need to parameterize breaking events on the basis of stratification and wave character alone without introducing the effects of the bottom boundary layer and bottom slope.

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